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Influence of nonlinear quantum dissipation on the dynamical properties of the f-deformed Jaynes-Cummings model in the dispersive limit

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Abstract. In this paper, we give a fully analytical description of the dynamics of an atom–field system, described by an f-deformed Jaynes-Cummings model, in the presence of nonlinear quantum dissipation and in the large detuning approximation. By solving analytically the f-deformed Liouville equation for the density operator at zero temperature, we explore the influence of nonlinear quantum dissipation on dynamical behavior of the atom-field system. Considering the field to be initially in a q-deformed coherent state, it is found that in the presence of nonlinear quantum dissipation (i) the amplitude of the entanglement between the field and the atom decreases with time, (ii) the sub-Poissonian characteristic of the initial cavity-field is enhanced at the initial stages of the evolution, but as time goes on the photon counting statistics asymptotically tends to the Poissonian statistics, and (iii) each of the two quadrature components of cavity-field exhibits damped oscillatory squeezing in the course of time and their quantum noises are asymptotically stabilized at the standard quantum limit.

PACS. 42.50.Ct Quantum description of interaction of light and matter; related experiments – 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements – 32.80.-t Photon interactions with atoms

1 Introduction

In quantum optics, one of the simplest and most nontrivial systems is the so-called Jaynes–Cummings model (JCM), which describes the interaction of a two-level atom with a single mode of the quantized electromagnetic field [1]. Investigations of the dynamical behavior of this model are extremely important due to its experimental realizations in high-Q microwave [2], in optical resonators [3] and in laser-cooled trapped ions [4]. Stimulated by the success of the JCM, more and more people have paid special attention to extending and generalizing the model in order to explore new quantum effects. Discussions related to several interesting generalizations of this model are now available in the literature [5] and the model is still promising in many applications, particularly in the fast developing research area of quantum information [6].

Among the generalized versions of the JCM the so-called f-deformed Jaynes-Cummings model (f-DJCM) has received much attention in view of its connection with quantum algebras [7]. In addition, it has been shown [8] that most of the nonlinear generalizations of the JCM are only particular cases of the f-DJCM. The quantum algebras, introduced as a mathematical description of deformed Lie algebras, have given the possibility of generalizing the notion of creation and annihilation operators of the usual quantum oscillator and to introduce deformed oscillator. Some deformed versions of oscillator algebra have found many applications to various physical problems, such as the algebraic treatment of quantum exactly solvable models [9], the bosonization of supersymmetric quantum mechanics [10], the treatment of vibrational spectra of molecules [11] and the investigation of nonlinearities in quantum optics [12]. The representation theory of the quantum algebras with a single deformation parameter q has led to the development of the q-deformed oscillator algebras [13]. Using a q-oscillator description Chaichian and co-workers [14] were the first to generalize the JCM Hamiltonian with an intensity-dependent coupling by relating it to the quantum $su_q(1,1)$ algebra. Similarly, Buzek [15] hoping to extract possible information about the physical meaning of the q-deformation, studied the atomic inversion of the standard JCM with a q-deformed field initially prepared in a *maths*- type q-deformed coherent state [16]. Bonatsos et al. [17] treated various versions of the JCM and their

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q-deformed extensions in a unified formalism based on the generalized deformed oscillator algebra. The quantum collapse and revival effects as well as the squeezing properties of the radiation field in the q-deformed version of the one-photon on-resonant JCM were investigated by Crnugelj et al. [18]. We have recently studied [19] the temporal evolution of atomic inversion and quantum fluctuations of atomic dipole variables in three variants of the two-photon q-DJCM for both on-and off-resonant atom-field interaction. Furthermore, by solving a dynamical problem characterized by a quite general f-DJCM, we have very recently proposed [20] a theoretical scheme to show the possibility of generating various families of nonlinear (f-deformed) coherent states [21] of the radiation field in a lossless coherently pumped micromaser.

However, all of the foregoing studies have been done only under the condition that the influence of the environment is not taken into account. The environment which is represented by a thermal reservoir always exists, and affects the system considered. No matter how weak the coupling to such an environment, the evolution of quantum subsystems is eventually affected by non-unitary features such as decoherence, dissipation and heating. From a mathematical point of view, the relevant state space, given by density matrices, has now a convex structure and the allowed quantum dynamics is described by completely positive maps. Initial pure states preparation are typically corrupted on extremely short time scales due to quantum coherence loss that turns them into mixed states [22]. The initial information irreversibly leaks out the system into the very large number of uncontrollable degrees of freedom of the environment. Over the last two decades much attention has been focused on the properties of the dissipative variants of the usual (non-deformed) JCM. The dissipative effects caused by the energy exchange between the system and environment have been studied both analytically [23] and numerically [24]. Last few years the JCM with phase damping (which occurs when there is no energy exchange between the system and environment), as applied to decoherence and entanglement, has been also studied intensively [25]. Furthermore, with the experimental realization of two-photon micromaser [26] the dissipative two-photon JCM has attracted a great deal of attention [27]. All of the above-cited studies have shown that dissipation affects markedly the dynamical behavior of the atom-field system.

In the present contribution our main purpose is to study the dynamical behavior of the atom-field system in the framework of the f-deformed dissipative JCM in the dispersive limit. Our model is based on the assumption that not only the atom-field interaction but also the coupling between the cavity-field and its environment is deformed. Formally, the model Hamiltonian for the atom-field system and the master equation for the reduced density operator of the damped field at zero temperature, respectively, have the same structure as non-deformed JCM and non-deformed master equation with the field operators $\hat{a}$ and $\hat{a}^+$ ($[\hat{a}, \hat{a}^+] = 1$) replaced by the deformed oscillator operators $\hat{A}$ and $\hat{A}^+$ obeying the f-deformed commutation relation $[\hat{A}, \hat{A}^+] = (\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n})$. The nonlinearity function $f(\hat{n})$ plays a central role in our treatment since it determines the form of nonlinearities of the field, the intensity-dependent atom-field coupling and the intensity-dependent field-reservoir coupling. With the field initially being in a q-deformed coherent state and the atom initially prepared in a coherent superposition of its ground and excited states, we investigate the influence of the deformed (nonlinear) dissipation at zero temperature on the atom-field entanglement, photon counting statistics and quadrature squeezing of the cavity field.

The paper is organized as follows. In Section 2, we introduce the theoretical model of dispersive interaction between a single-mode cavity-field and a two-level atom within the framework of an f-DJCM with an arbitrary nonlinearity function $f(\hat{n})$. In Section 3, we give an analytic solution for the f-deformed master equation describing the f-deformed interaction of the system "atom + field mode" with a zero temperature reservoir. In Section 4, we employ the analytic results obtained in Section 3 to investigate the influence of nonlinear (f-deformed) dissipation on the dynamical properties of the f-DJCM. Finally, we summarize our conclusions in Section 5.

2 The dispersive f-deformed JCM

In this section, we consider a deformed single-mode field interacting with an effective two-level atom without considering the influence of the dissipation. The Hamiltonian of the system under the rotating wave approximation has the following form

$$\hat{H} = \omega \hat{A}^+\hat{A} + 1/2\omega_{eg}\ \hat{\sigma}_z + g\left(\hat{A}^+\hat{\sigma}^- + \hat{A}\hat{\sigma}^+\right) \quad (\hbar = 1), \tag{1}$$

where the two atomic levels $|e\rangle$ (excited state) and $|g\rangle$ (ground state) separated by an energy difference ω_{eg} are represented by the Pauli matrices, the coupling constant g is a real number and ω is the frequency of the field. The operators $\hat{A}$ and $\hat{A}^+$ are the f-deformed annihilation and creation operators constructed from the usual bosonic operators $\hat{a}, \hat{a}^+$ ($[\hat{a}, \hat{a}^+] = 1$) and number operator $\hat{n} = \hat{a}^+\hat{a}$ as $\hat{A} = \hat{a}f(\hat{n})$ and $\hat{A}^+ = f(\hat{n})\hat{a}^+$, in which $f(\hat{n})$ is an arbitrary real function of $\hat{n}$. The deformed operators $\hat{A}$, $\hat{A}^+$ satisfy the f-deformed bosonic oscillator commutation relations

$$[\hat{A}, \hat{A}^+] = (\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n}),$$
$$[\hat{A}, \hat{n}] = \hat{A},\ [\hat{A}^+, \hat{n}] = -\hat{A}^+. \tag{2}$$

It is evident that in the limiting case $f(\hat{n}) = 1$, the Hamiltonian (1) becomes the conventional (non-deformed) JC Hamiltonian and the algebra (2) reduces to the well-known Heisenberg-Weyl algebra generated by $\hat{a}$, $\hat{a}^+$ and the identity $\hat{I}$. The f-DJCM given by (1) is of considerable interest because of its relevance to the study of the intensity-dependent interaction between a single atom and the radiation field in quantum optics [28] as well as the study of the quantized motion of a single ion in an anharmonic-oscillator potential trap [29]. It has been shown [19] that the above Hamiltonian describes an intensity–dependent coupling between a single two-level atom and a non-deformed single-mode radiation field in the presence of an additional nonlinear interaction. As a well-known example if we choose $f(\hat{n}) = \sqrt{1 + k(\hat{n} - 1)}$, where k is a positive constant, the model consists of a single two-level atom interacting through an intensity-dependent coupling with a single–mode field surrounded by a nonlinear Kerr-like medium contained inside a lossless cavity. Physically, this model may be realized as if the cavity contains two different species of Rydberg atoms, of which one behaves like a two-level atom and the other behaves like an anharmonic oscillator in the single mode field of frequency ω [30].

The eigenvalues and eigenstates of the Hamiltonian (1) are respectively given by [19]

$$E_{\pm,n} = \frac{\omega}{2}\left((n+1)f^2(n+1) + nf^2(n)\right) \pm \frac{1}{2}\sqrt{\Delta_n^2 + 4g^2(n+1)f^2(n+1)}, \tag{3a}$$

$$|+,n\rangle = \cos\vartheta_n\,|e,n\rangle + \sin\vartheta_n\,|g,n+1\rangle\,, \tag{3b}$$

$$|-,n\rangle = \sin\vartheta_n\,|e,n\rangle - \cos\vartheta_n\,|g,n+1\rangle\,, \tag{3c}$$

where by definition

$$\Delta_n \equiv \Delta - \omega\left((n+1)f^2(n+1) - nf^2(n) - 1\right), \quad (\Delta \equiv \omega_{eg} - \omega) \tag{4a}$$

$$\cos\vartheta_n \equiv \frac{2g\sqrt{(n+1)f^2(n+1)}}{\sqrt{\left(\Omega_n - \Delta_n\right)^2 + 4g^2(n+1)f^2(n+1)}}, \tag{4b}$$

$$\sin\vartheta_n \equiv \frac{(\Omega_n - \Delta_n)}{\sqrt{\left(\Omega_n - \Delta_n\right)^2 + 4g^2(n+1)f^2(n+1)}}, \tag{4c}$$

with

$$\Omega_n \equiv \sqrt{\Delta_n^2 + 4g^2(n+1)f^2(n+1)}, \tag{4d}$$

as the f-deformed Rabi frequency. Depending on the form of $f(\hat{n})$ the function Ω_n has a different behavior in comparison with the conventional Rabi frequency. For example, in the nondeformed case, $f(\hat{n}) = 1$, the minimum of energy separation $\Delta E_n \equiv E_{+,n} - E_{-,n}$ occurs at $\Delta = 0$, while for $f(\hat{n}) \neq 1$ the value of Δ at which the minimum separation of eigenenergies occurs is shifted [19].

Following Peixoto et al. [31], in the large detuning approximation, that is

$$\omega_{eg} - \omega \gg \omega\left((n+1)f^2(n+1) - nf^2(n) - 1\right) + \sqrt{g^2(n+1)f^2(n+1)}, \tag{5}$$

for any "relevant" photon number, we arrive at the f-deformed effective Hamiltonian

$$\hat{H}_{\mathit{eff}} = \omega\hat{A}^+\hat{A} + 1/2\,\omega_{eg}\hat{\sigma}_z + \hat{H}_{\mathit{eff}}^{(I)}, \tag{6a}$$

$$\hat{H}_{\mathit{eff}}^{(I)} \equiv k\left(A\hat{A}^+\,|e\rangle\,\langle e| - \hat{A}^+\hat{A}\,|g\rangle\,\langle g|\right), \tag{6b}$$

with $k \equiv g^2/\Delta$. The above effective Hamiltonian which does not cause any transition in the system creates an entanglement between the atomic and the field states. For a dispersive non-deformed JCM this entanglement has been studied in detail and it has been used extensively in the context of the generation of Schrödinger cats and atom optics in quantized light fields [32].

3 The f-deformed master equation and its analytic solution

We now consider the f-deformed field interacting dispersively with a single two-level atom and coupled to a zero temperature reservoir. Recently, Isar et al. [33] have derived a master equation for the f-deformed harmonic oscillator in the presence of a dissipative environment, for the case of an f-deformed interaction of the oscillator with its environment. Considering the environment as a thermal bath at equilibrium temperature T, the master equation for the damped f-deformed oscillator in the interaction picture has the following form under the Born-Markov approximation [32]

$$\frac{d}{dt}\hat{\rho}_{D.O}(t) = \frac{\lambda}{2}\left(\left[\left[\coth\frac{\hbar\omega\Omega(\hat{n})}{2k_BT}\hat{A}, \hat{\rho}_{D.O}(t)\right], \hat{A}^+\right] - \left[\hat{A}^+, \left\{\hat{A}, \hat{\rho}_{D.O}(t)\right\}\right] + h.c.\right), \tag{7}$$

where, $\hat{\rho}_{D.O}$ is the reduced density operator for the f-deformed oscillator with frequency ω, λ is the dissipation constant, k_B is the Boltzman constant, and by definition

$$\Omega(\hat{n}) \equiv \frac{1}{2}\left((\hat{n}+2)f^2(\hat{n}+2) - \hat{n}f^2(\hat{n})\right). \tag{8}$$

Furthermore, in equation (7) the notation { , } stands for anticommutator. Equation (7) is the f-deformed version of the master equation for a conventional (non-deformed) damped harmonic oscillator obtained in the framework of the Lindblad theory for open quantum systems [34]. If the bath temperature is $T = 0$, the master equation (7) simplifies

$$\frac{d}{dt}\hat{\rho}_{D.O}(t) = -\lambda\left(\hat{A}^+\hat{A}\hat{\rho}_{D.O}(t) + \hat{\rho}_{D.O}(t)\hat{A}^+\hat{A} - 2\hat{A}\hat{\rho}_{D.O}(t)\hat{A}^+\right). \tag{9}$$

Therefore, the evolution of the compound atom-field system in a dispersive f-deformed JCM and in the presence of the f-deformed (nonlinear) dissipation at zero temperature can be written in the interaction picture as

$$\frac{d}{dt}\hat{\rho}(t) = -i\left[\hat{H}_{\mathit{eff}}^{(I)}, \hat{\rho}(t)\right] - L_{nl}\hat{\rho}(t), \quad (\hbar = 1), \tag{10}$$

where $\hat{\rho}$ is the atom-field density operator, $\hat{H}_{\mathit{eff}}^{(I)}$ is given by equation (6b) and the nonlinear (f-deformed) superoperator L_{nl} is defined as

$$\begin{aligned}L_{nl}. &\equiv \lambda\left(\hat{A}^{+}\hat{A}\;.\;+\;.\hat{A}^{+}\hat{A}-2\hat{A}.\;\hat{A}^{+}\right)\\&=\lambda\left(\hat{a}^{+}\hat{a}f\;(\hat{a}^{+}\hat{a}).\;+\;.\hat{a}^{+}\hat{a}f(\hat{a}^{+}\hat{a})\right.\\&\quad\left.-\,2\hat{a}f(\hat{a}^{+}\hat{a}).\;f(\hat{a}^{+}\hat{a})\hat{a}^{+}\right),\end{aligned} \tag{11}$$

containing the non-unitary contributions to the dynamics of $\hat{\rho}(t)$. This superoperator is a linear combination of bosonic superoperators [31,35], which form a finite Lie algebra under commutation. The bosonic superoperators represent the action of creation and annihilation operators of the harmonic oscillator on an operator $\hat{O}$:

$$\begin{aligned}a^{\ell}\hat{O}\equiv\hat{a}\hat{O},\quad \left(a^{\ell}\right)^{+}\hat{O}\equiv\hat{a}^{+}\hat{O},\\ a^{r}\hat{O}\equiv\hat{O}\hat{a},\quad \left(a^{r}\right)^{+}\hat{O}\equiv\hat{O}\hat{a}^{+}.\end{aligned} \tag{12}$$

The sets $\left(a^{\ell},(a^{\ell})^{+},1\right)$ and $(a^{r},(a^{r})^{+},1)$ constitute left and right realizations of the Heisenberg-Weyl group $hw(4)$ [36], denoted $hw_{l}(4)$ and $hw_{r}(4)$, respectively. From the fundamental commutation relation $[\hat{a},\ \hat{a}^{+}]=1$ and the definitions given by (12), one can derive the commutation relations between the bosonic superoperators

$$\left[a^{\ell},\left(a^{\ell}\right)^{+}\right]=1,\quad\left[a^{r},(a^{r})^{+}\right]=-1. \tag{13}$$

A superoperator belonging to $hw_{l}(4)$ commutes with another belonging to $hw_{r}(4)$. The bilinear products of these superoperators are

$$M\equiv\left(a^{\ell}\right)^{+}a^{\ell},\;\;P\equiv a^{r}\left(a^{r}\right)^{+},\;\;J\equiv a^{\ell}\left(a^{r}\right)^{+}=\left(a^{r}\right)^{+}a^{\ell}. \tag{14}$$

The above defined superoperators generate a finite Lie algebra. It is easy to show that

$$\begin{aligned}&[J,M]=J,\quad [J,P]=J,\quad [M,P]=0,\\&[J,f^{2}(M)]=J\left(f^{2}(M)-f^{2}(M-1)\right),\\&[J,f^{2}(P)]=J\left(f^{2}(P)-f^{2}(P-1)\right).\end{aligned} \tag{15}$$

Now the master equation (10) can be solved by applying the dynamical symmetry method proposed in reference [37]. For this purpose, we assume that the two-level atom is initially prepared in a coherent superposition of the excited state $|e\rangle$ and the ground state $|g\rangle$,

$$\hat{\rho}_{a}(0)=|\psi\rangle_{a}\,{}_{a}\langle\psi|=\sum_{i,j=e,g}c_{i}c_{j}^{*}\,|i\rangle\,\langle j|\,,\quad |c_{e}|^{2}+|c_{g}|^{2}=1, \tag{16}$$

and the field is initially in a nonlinear (f-deformed) coherent state $|z\rangle_{f}$,

$$\hat{\rho}_{f}(0)=|z\rangle_{f}\,{}_{f}\langle z|=\sum_{m,n=0}^{\infty}Q_{n}(z)Q_{m}^{*}(z)\,|n\rangle\,\langle m|, \tag{17}$$

where $Q_{n}(z)\equiv Nz^{n}/\sqrt{(nf^{2}(n))}!$ (N, normalization constant) and $z\equiv|z|e^{i\psi}$. The states $|z\rangle_{f}$ are defined as right eigenstates of the f-deformed annihilation operator $\hat{A}=\hat{a}f(\hat{N})$, i.e. $\hat{A}\,|z\rangle_{f}=z\,|z\rangle_{f}$. Suppose that at the moment the interaction between the atom and the field starts, the state of the atom-field system is the direct product of equations (16) and (17), $\hat{\rho}(0)=\hat{\rho}_{a}(0)\otimes\hat{\rho}_{f}(0)$. After some lengthy but straightforward calculation we obtain the following analytical expressions for the matrix elements of the atom-field density operator $\hat{\rho}_{ij}(t)$ $(i,j=e,g)$,

$$\begin{aligned}\hat{\rho}_{ee}(t)=|c_{e}|^{2}&\sum_{n,m=0}^{\infty}Q_{n}(z)Q_{m}^{*}(z)\\&\times\exp\left(\mathit{\Gamma}_{ee}(A(m,n),B(m,n),t)\right.\\&\left.+\,i\mathit{\Phi}_{ee}(A(m,n),B(m,n),t)\right)|n\rangle\,\langle m|\,,\end{aligned} \tag{18a}$$

$$\begin{aligned}\hat{\rho}_{gg}(t)=|c_{g}|^{2}&\sum_{n,m=0}^{\infty}Q_{n}(z)Q_{m}^{*}(z)\\&\times\exp\left(\mathit{\Gamma}_{ee}(A(m-1,n-1),B(m,n),t)\right.\\&\left.+\,i\mathit{\Phi}_{ee}(A(m-1,n-1),B(m,n),t)\right)|n\rangle\,\langle m|\,,\end{aligned} \tag{18b}$$

$$\begin{aligned}\hat{\rho}_{eg}(t)=c_{e}^{*}c_{g}&\sum_{n,m=0}^{\infty}Q_{n}(z)Q_{m}^{*}(z)\\&\times\exp\left(\mathit{\Gamma}_{eg}(B(m,n),t)+i\mathit{\Phi}_{eg}(B(m,n),t)\right)|n\rangle\,\langle m|\,,\end{aligned} \tag{18c}$$

$$\begin{aligned}\hat{\rho}_{ge}(t)=c_{e}c_{g}^{*}&\sum_{n,m=0}^{\infty}Q_{n}(z)Q_{m}^{*}(z)\\&\times\exp(\mathit{\Gamma}_{eg}(B(n,m),t)-i\mathit{\Phi}_{eg}(B(n,m),t))\,|n\rangle\,\langle m|\,,\end{aligned} \tag{18d}$$

where

$$\begin{aligned}\mathit{\Gamma}_{ee}(A(m,n),B(m,n),t)&=-\lambda t\left(mf^{2}(m)+nf^{2}(n)\right)\\&+\frac{2\lambda|z|^{2}}{\lambda^{2}B^{2}(m+1,n+1)+k^{2}A^{2}(m+1,n+1)}\\&\times\Big\{\lambda B(m+1,n+1)-e^{-\lambda tB(m+1,n+1)}\\&\times[\lambda B(m+1,n+1)\cos(ktA(m+1,n+1))\\&-kA(m+1,n+1)\sin(ktA(m+1,n+1))]\Big\},\end{aligned} \tag{19a}$$

$$\begin{aligned}\mathit{\Phi}_{ee}(A(m,n),B(m,n),t)&=kt\left((m+1)f^{2}(m+1)\right.\\&\left.-(n+1)f^{2}(n+1)\right)\\&-\frac{2\lambda|z|^{2}}{\lambda^{2}B^{2}(m+1,n+1)+k^{2}A^{2}(m+1,n+1)}\\&\times\Big\{kA(m+1,n+1)-e^{-\lambda tB(m+1,n+1)}\\&[(kA(m+1,n+1)\cos(ktA(m+1,n+1))\\&+kB(m+1,n+1)\sin(ktA(m+1,n+1))]\Big\},\end{aligned} \tag{19b}$$

$$\begin{aligned}\Gamma_{eg}(B(m,n),t) &= -\lambda t\left(mf^2(m)+nf^2(n)\right)\\ &+\frac{2\lambda|z|^2}{\lambda^2B^2(m+1,n+1)+k^2B^2(m+2,n+1)}\\ &\times\Big\{\lambda B(m+1,n+1)-e^{-\lambda tB(m+1,n+1)}\\ &\times[\lambda B(m+1,n+1)\cos(ktB(m+2,n+1))\\ &-kB(m+2,n+1)\sin(ktB(m+2,n+1))]\Big\}, \quad (19c)\end{aligned}$$

$$\begin{aligned}\Phi_{eg}(B(m,n),t) &= -kt\left((m+1)f^2(m+1)\right.\\ &\left.+(n+1)f^2(n+1)\right)\\ &-\frac{2\lambda|z|^2}{\lambda^2B^2(m+1,n+1)+k^2B^2(m+2,n+1)}\\ &\times\Big\{kB(m+2,n+1)-e^{-\lambda tB(m+1,n+1)}\\ &\times[(kB(m+2,n+1)\cos(ktB(m+2,n+1))\\ &+\lambda B(m+1,n+1)\sin(ktB(m+2,n+1))]\Big\}, \quad (19d)\end{aligned}$$

together with

$$\begin{aligned}A(m,n) &= (m+1)f^2(m+1)-mf^2(m)\\ &-(n+1)f^2(n+1)+nf^2(n), \quad (19e)\\ B(m,n) &= mf^2(m)-(m-1)f^2(m-1)\\ &+(n+1)f^2(n+1)-nf^2(n). \quad (19f)\end{aligned}$$

Making use of the solution given by (18), one can evaluate the mean values of operators of interest. In the next section we shall use it to investigate various dynamical properties of the dissipative f-DJCM in the dispersive approximation.

4 Dynamical properties of the model

In this section, we study the influence of the f-deformed dissipation on the time evolution of various properties of the f-DJCM, particularly its non-classical features.

4.1 Linear entropies and atom-field entanglement

It is well-known that if the field and the atom in the JCM are initially prepared in a pure state, then at $t > 0$ the atom-field system evolves into an entangled state. In this entangled state, the field and the atom separately are in mixed states. The entanglement between the atom and the field, as well as the decoherence induced by the cavity of the usual dispersive JCM was studied in reference [31]. It has been shown that the cavity has practically no influence on the coherence properties of the field from a qualitative point of view. However, although the atom is not directly coupled to the cavity-field, its coherence properties are strongly influenced by dissipation both qualitatively and quantitatively.

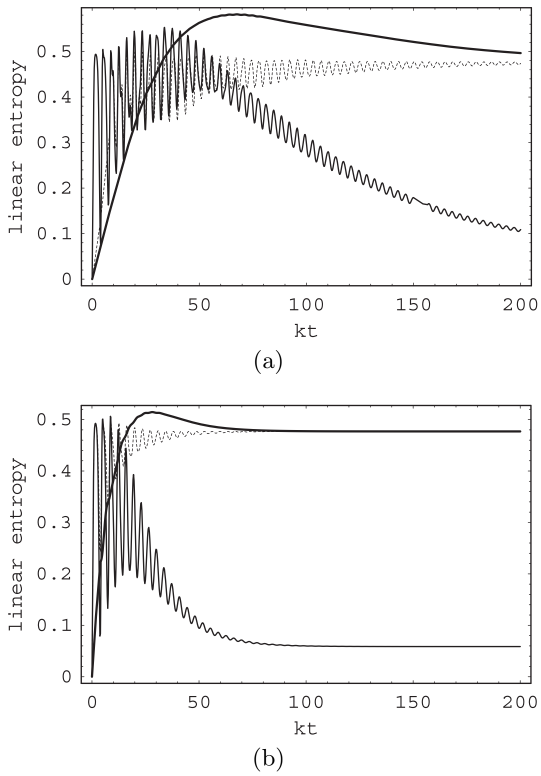

Fig. 1. Time evolution of the linear entropies $s_{a-f}(t)$ (——), $s_a(t)$ (- - - -) and $s_f(t)$ (——) as functions of the scaled time kt, for $f(n) = \sqrt{(q^n-1)/[n(q-1)]}$, $q = 0.9$, $c_e = c_g = 1/\sqrt{2}$, $|z|^2 = 5$ and for different values of the dissipation constant; (a) $\lambda/k = 0.05$, (b) $\lambda/k = 0.1$.

The stability of quantum coherence may be understood as the process where quantum coherence of the state of a physical system is preserved along its time evolution. In this sense we say that an initial pure quantum state, described by a density operator $\hat{\rho}$, is "stable" if $\mathrm{Tr}\,\hat{\rho}^2 = 1$, for all times. One way to measure decoherence, or the stability of an initial pure state is to use the linear entropy [38] $s = 1 - \mathrm{Tr}\,\hat{\rho}^2$. The time evolution of the atomic (field) entropy reflects the time evolution of the degree of entanglement between the atom and the field. The higher the entropy is, the greater the entanglement between the atom and the field becomes.

In the non-deformed dispersive JCM [31] only the coherence of the atom is influenced by the cavity, though the atom does not couple to the cavity directly. The coherence of the field remains unchanged by the environment and the linear entropy of the field behaves periodically, with a maximum value 0.5 and a minimum value 0, which correspond to the entanglement and disentanglement between the field and the atom, respectively. However, in the f-DJCM we considered, the field is also affected by the cavity and its coherence will lose due to the nonlinear dissipation. In Figures 1a and 1b we display the influence of nonlinear dissipation on time evolution of the linear entropies of the atom-filed system, $s_{a-f}(t) = 1 - \mathrm{Tr}\left(\hat{\rho}^2(t)\right)$, of the field, $s_f(t) = 1 - \mathrm{Tr}_f(\hat{\rho}_f^2(t))$, and of the atom,

$s_a(t) = 1 - \mathrm{Tr}_a\left(\hat{\rho}_a^2(t)\right)$, as functions of the scaled time kt, in the special case $f(n) = \sqrt{(q^n - 1)/[n(q-1)]}$ $(0 < q < \infty)$. The deformation parameter q may be viewed as a phenomenological constant controlling the strength of the intensity-dependent coupling between the atom and the field as well as between the field and the reservoir. Furthermore, this choice of nonlinearity function $f(n)$ corresponds to the maths-type q-deformed coherent state [16] as the initial state of the cavity-field. As it is seen, both the field and the atom linear entropies have damped oscillations. The presence of the local maxima and minima in the temporal evolution of $s_a(t)$ and $s_f(t)$ is due to the entanglement and disentanglement between the field and the atom. Because of the influence of dissipation on the entanglement, the amplitude of the entanglement decreases with the time. During the repeating periods of entanglement and disentanglement, the field loses and gains its coherence. One finds that the larger λ is, the more quickly the amplitude is suppressed, and the more rapidly $s_{a-f}(t), s_f(t)$ and $s_a(t)$ reach their maximum values and asymptotic values.

4.2 Photon counting statistics

One of the most remarkable non-classical effect is the sub-Poissonian photon statistics of the field state. To determine such effect we consider the Mandel Q parameter defined by [39]

$$Q(t) = \frac{\left(\langle \hat{n}(t)^2 \rangle - \langle \hat{n}(t) \rangle^2\right) - \langle \hat{n}(t) \rangle}{\langle \hat{n}(t) \rangle}$$
$$= \frac{\left(\mathrm{Tr}_f(\hat{\rho}_f(t)\hat{n}^2) - (\mathrm{Tr}_f(\hat{\rho}_f(t)\hat{n}))^2\right) - \mathrm{Tr}_f(\hat{\rho}_f(t)\hat{n})}{\mathrm{Tr}_f(\hat{\rho}_f(t)\hat{n})}. \quad (20)$$

For $Q < 0$ $(Q > 0)$, the statistics is sub-Poissonian (super-Poissonian); $Q = 0$ stands for Poissonian statistics. In Figure 2a we have plotted Mandel Q parameter versus the scaled time kt for the same corresponding data used in Figures 1 but for some different values of λ/k. As it is seen, at the initial stages of evolution the sub-Poissonian characteristic of the cavity-field is enhanced (note that the initial cavity-field, i.e. maths-type q-deformed coherent state, is sub-Poissonian for $q < 1$). However, as time goes on, the Mandel parameter increases and it is finally stabilized at an asymptotical zero value corresponding to the Poissonian statistics. The rates with which the enhancement of sub-Poissonian statistics and reaching the Poissonian statistics occur are directly proportional to the dissipation constant λ; the greater λ is, the more quickly the Mandel parameter decreases, and the more rapidly it tends to the asymptotic value.

It is worth noticing that in the case of usual (non-deformed) dissipative and dispersive JCM, i.e. for $f(\hat{n}) = 1$, at the initial stages of evolution the Mandel parameter increases in comparison with its initial value $(Q = 0)$ which shows super-Poissonian statistics. As time goes on, this parameter decreases and finally tends to Poissonian statistics (see Fig. 2b).

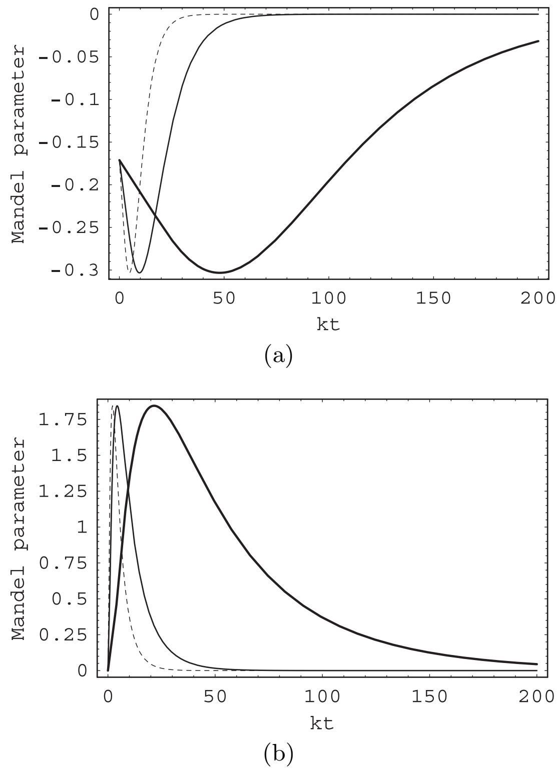

Fig. 2. Time evolution of the Mandel parameter as a function of the scaled time kt, for $c_e = c_g = 1/\sqrt{2}$, $|z|^2 = 5$; (a) $f(n) = \sqrt{(q^n - 1)/[n(q-1)]}$, $q = 0.9$; (——): $\lambda/k = 0.05$, (——): $\lambda/k = 0.1$, (- - - -): $\lambda/k = 0.2$, (b) $f(n) = 1$; (——): $\lambda/k = 0.05$, (——): $\lambda/k = 0.1$, (- - - -) : $\lambda/k = 0.2$.

4.3 Quadrature squeezing of the cavity-field

The experiments on photon antibunching and sub-Poissonian statistics have concerned with the intensity or photon-number fluctuations of electromagnetic field. Latter, there was a major studies focused on the fluctuations in the quadrature amplitudes of the electromagnetic field to produce squeezed light. This light is indicated by having less noise in one field quadrature than vacuum state with an excess of noise in the conjugate quadrature such that the product of canonically conjugate variances must satisfy the uncertainty relation. In the studies of quantum optics theory, this light occupies a wide area because of its various applications, e.g., in optical communication networks [40], in interferometric techniques [41], and in optical waveguide tap [42]. Furthermore, investigation of the squeezing properties of the radiation field is a central topic in quantum optics and noise squeezing can be measured by means of homodyne detection [43].

In order to investigate the quadrature squeezing of the dissipative f-DJCM in the presence of nonlinear cavity damping, we introduce the two slowly varying Hermitian f-deformed quadrature components $\hat{X}_{1A}$ and $\hat{X}_{2A}$ defined by, respectively,

$$\hat{X}_{1A}(t) \equiv \frac{1}{2}(\hat{A}e^{i\omega t} + \hat{A}^{+}e^{-i\omega t}),$$
$$\hat{X}_{2A}(t) \equiv \frac{1}{2i}(\hat{A}e^{i\omega t} - \hat{A}^{+}e^{-i\omega t}). \quad (21)$$

In the limiting case $f(\hat{n}) = 1$, these two operators reduce to the conventional (non-deformed) quadrature operators [43]. The commutation of $\hat{X}_{1A}(t)$ and $\hat{X}_{2A}(t)$ is

$$[\hat{X}_{1A}(t), \hat{X}_{2A}(t)] = \frac{i}{2}\left((\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n})\right). \quad (22)$$

The variances $\langle(\Delta\hat{X}_{iA}(t))^2\rangle \equiv \langle\hat{X}_{iA}^2(t)\rangle - \langle\hat{X}_{iA}(t)\rangle^2$ $(i = 1, 2)$ satisfy the uncertainty relation

$$\langle(\Delta\hat{X}_{1A}(t))^2\rangle\langle(\Delta\hat{X}_{2A}(t))^2\rangle \geq \frac{1}{16}(\langle(\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n})\rangle)^2. \quad (23)$$

A state of the field is said to be squeezed when one of the quadrature components $\hat{X}_{1A}(t)$ and $\hat{X}_{2A}(t)$ satisfies the relation

$$\left\langle(\Delta\hat{X}_{iA}(t))^2\right\rangle < \frac{1}{4}\left(\langle(\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n})\rangle\right) \quad (i = 1 \text{ or } 2). \quad (24)$$

The degree of squeezing can be measured by the squeezing parameter $S_i (i = 1, 2)$ defined by

$$S_i(t) \equiv 4\langle(\Delta\hat{X}_{iA}(t))^2\rangle - (\langle(\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n})\rangle), \quad (25)$$

which can be expressed in terms of the f-deformed annihilation and creation operators of the field as follows

$$S_1(t) = 2B_0(t) + 2\mathrm{Re}(B_2(t)) - (2\mathrm{Re}(B_1(t)))^2, \quad (26a)$$
$$S_2(t) = 2B_0(t) - 2\mathrm{Re}(B_2(t)) - (2\mathrm{Im}(B_1(t)))^2, \quad (26b)$$

where

$$B_0(t) \equiv \langle\hat{A}^{+}\hat{A}\rangle, \quad B_1(t) \equiv \langle\hat{A}\rangle e^{i\omega t}, \quad B_2(t) \equiv \langle\hat{A}^2\rangle e^{2i\omega t}. \quad (26c)$$

Then, the condition for squeezing in the quadrature component can be simply written as $S_i(t) < 0$.

In Figures 3a and 3b we have plotted the squeezing parameters $S_i(t)$ $(i = 1, 2)$ versus the scaled time kt for the same corresponding data, respectively, used in Figures 1a and 1b. As it is seen, $S_1(t)$ and $S_2(t)$ show damped oscillatory behavior and each of the two quadrature components exhibits squeezing in the course of time evolution. Because of the influence of nonlinear dissipation, the amplitude of the squeezing decreases with the time. Furthermore, with the increasing value of dissipation constant, the amplitude of the oscillations of $S_1(t)$ and $S_2(t)$ is further suppressed, and the quadrature squeezing becomes weaker. After some time, $S_1(t)$ and $S_2(t)$ are stabilized at an asymptotical zero value; the larger the dissipation constant λ is, the more rapidly $S_1(t)$ and $S_2(t)$ reach the asymptotic value zero.

It is noticeable that in the case of usual (non-deformed) dissipative and dispersive JCM, i.e. for $f(\hat{n}) = 1$, neither $\hat{X}_{1A}$ nor $\hat{X}_{2A}$ exhibits squeezing (see Fig. 3c).

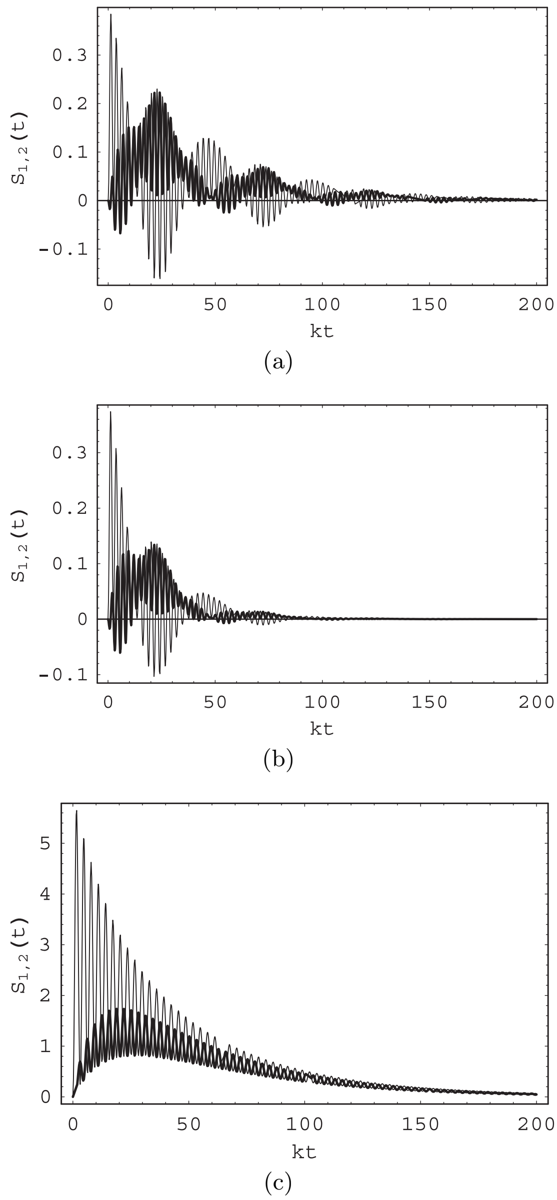

Fig. 3. Time evolution of $S_1(t)$ (——) and $S_2(t)$ (——) as functions of the scaled time kt, for $c_e = c_g = 1/\sqrt{2}$, $|z|^2 = 5$; (a) $f(n) = \sqrt{(q^n-1)/[n(q-1)]}$, $q = 0.9$, $\lambda/k = 0.05$, (b) $f(n) = \sqrt{(q^n-1)/[n(q-1)]}$, $q = 0.9$, $\lambda/k = 0.1$, (c) $f(n) = 1$, $\lambda/k = 0.05$.

5 Summary and conclusions

Our purpose was to study the dynamical properties of an atom–field system, described by an f-DJCM, in a deformed interaction with a dissipative environment, modeled as a thermal bath at zero temperature. In the large detuning approximation, by applying the dynamical symmetry method, we solved the f-deformed master equation to obtain analytical expressions for the matrix elements of the atom-field density operator. Whilst the model was quite general, we looked specifically at a special choice of the nonlinearity, i.e. q-nonlinearity. Considering the cavity-field to be initially in a q-deformed coherent state and the two-level atom to be in a coherent superposition of its excited and ground states, we investigated the influence of the deformed (nonlinear) dissipation at zero temperature on the atom-field entanglement, photon counting statistics and quadrature squeezing of the cavity-field. We found: (1) both the field and the atom linear entropies have damped oscillations and due to the influence of dissipation on the entanglement, the amplitude of the entanglement decreases with the time. The larger dissipation constant λ is, the more quickly the amplitude is suppressed, and the more rapidly the linear entropies reach their maximum values and asymptotic values. (2) At the initial stages of evolution the sub-Poissonian characteristic of the cavity-field is enhanced. However, as time goes on, the Mandel parameter increases and it is finally stabilized at an asymptotical zero value corresponding to the Poissonian statistics; the greater λ is, the more quickly the Mandel parameter decreases, and the more rapidly it tends to the asymptotic value. (3) The cavity-field quadratures show damped oscillatory behavior and each of the two quadrature components exhibits squeezing in the course of time evolution. The larger λ is, the more rapidly quadrature components reach the asymptotic value zero.

The model presented here has not only demonstrated the basic features of the dissipative dynamics of an f-deformed Jaynes-Cummings model theoretically, but is of experimental importance in measuring the observable quantities of the atom-field system in a dissipative cavity containing any kind of nonlinearities in the future, providing some guidelines to experimentalists in the identification of the kind of unknown nonlinear medium and the utilization of the nonlinearities of dissipative f-DJCM.

It is noticeable that the dispersive f-DJCM may be used in the context of the generation of the so-called f-deformed Schrödinger cat states. Therefore, it is expected that our treatment may be applied to investigate the dissipative dynamics of those states and the related coherence properties. We hope to report on such issue in a forthcoming paper.

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